Magnetic Field Decay and Period Evolution of Anomalous X-Ray Pulsars in the Context of Quark Stars

Brian Niebergal, Rachid Ouyed and Denis Leahy

Department of Physics and Astronomy, University of Calgary, 2500 University Drive NW, Calgary, Alberta, T2N 1N4 Canada

bnieber@iras.ucalgary.ca

ABSTRACT

We discuss a model wherein soft gamma-ray repeaters (SGRs), anomalous X-ray pulsars (AXPs), and radio quiet isolated neutron stars (RQINSs) are all compact objects exhibiting superconductivity, namely color-flavor locked quark stars. In particular we calculate the magnetic field decay due to the expulsion of spin-induced vortices from the star's superfluid-superconducting interior, and the resultant spin-down rate. We find that, for initial parameters characteristic of AXPs/SGRs ($10^{13} < B < 10^{14}$ G; $3 < P < 12$ s), the magnetic field strengths and periods remain unchanged within a factor of two for timescales of the order of $5\times10^5 - 5\times10^7$ yrs given a quark star of radius 10 km. Within these timescales, we show that the observed period clustering in RQINSs can be explained by compactness, as well as calculate how the magnetic field and period evolve in a manner concurrent with RQINS observations.

Subject headings: gamma rays: bursts — X-rays: stars — stars: magnetic fields — stars: neutron — dense matter

1. Introduction

Soft γ-ray repeaters (SGRs) are sources of recurrent, short ($t \sim 0.1\,\mathrm{s}$), intense ($L \sim 10^{44}$ ergs) bursts of γ-ray emission with a soft energy spectrum. The normal pattern of SGRs are intense activity periods which can last weeks or months, separated by quiescent phases lasting years or decades. The three most intense SGR bursts ever recorded were the 5 March 1979 giant flare of SGR 0526-66 (Mazets et al. 1979), the similar 28 August 1998 giant flare of SGR 1900+14 and the 27 December 2004 burst (SGR 1806-20). AXPs are similar in nature but with a somewhat weaker intensity and no recurrent bursting. Several

SGRs/AXPs have been found to be X-ray pulsars with an unusually high spin-down rate of $\dot{P}/P \sim 10^{-10}$ s^{-1}, usually attributed to magnetic braking caused by a super-strong magnetic field.

The model normally reserved for SGRs/AXPs is the magnetar model, however it has been suggested (Ouyed et al. 2004) that CFL (color-flavor locked) quark stars could be also responsible for their activity. In this quark star model, we assume a neutron star has made the transition from hadronic to superfluid-superconducting CFL quark matter. Through the Meissner effect, the quark star's interior magnetic field is forced inside rotationally induced vortices that are aligned with the rotation axis of the star. The exterior dipole field is forced to align with the rotation axis as simulated in Ouyed et al.(2005), with application to SGRs/AXPs.

In this Letter, we extend the model by studying the post-alignment spin-down phase. Since the star's interior is a superconducting-superfluid, the number of vortices contained within has a quantized relation to the spin period. As the star spins down, the vortices containing the magnetic field are forced to the surface where they are expelled. The contained magnetic field decays through reconnection events, thus lowering the spin-down rate and providing X-ray emission. Our model bears some similarities to the picture presented in Srinivasan et al. (1990) for the case of neutron stars where vortex expulsion is intimately related to the rotation history of the neutron star. What makes our model work is the the fact that the fluxoids are trapped inside the vortices, and the absence of a crust (absence of electrons) which removes any pinning forces.

Using this model of vortex expulsion, along with simple magnetic dipolar braking, we calculate the period evolution and magnetic field decay of the quark star. We also show how the evolution of quark stars (AXPs/SGRs) leads to parameters indicative of radio quiet isolated neutron stars (RQINSs; see § 4). We then show how the compactness of a quark star governs the period clustering in RQINSs.

The paper is presented as follows: In § 2 we describe the formation of quark stars and how the Meissner effect constrains the magnetic field into vortices aligned with the rotation axis. We then calculate the magnetic field decay and period evolution during the quiescent phase in § 3. Then, in § 4, we show how our model predicts magnetic field strengths, periods, and ages consistent with RQINS observations, and, discuss the results in the context of a $P - \dot{P}$ diagram. Finally in § 4, we describe the relation between the compactness of the quark star and the resultant period clustering. We conclude in § 6.

2. Foundation of the Model

Assume a quark star is born with a temperature $T > T_{\rm c}$ ($T_{\rm c}$ is the critical temperature below which superconductivity sets in), and enters a superconducting-superfluid phase in the core as it cools by neutrino emission (Ouyed et al. 2002; Keränen et al. 2005), and contracts due to spin-down. The front quickly expands to the entire star followed by the formation of rotationally induced vortices, analogous to rotating superfluid ^{3}He (the vortex lines are parallel to the rotation axis; Tilley&Tilley 1990). Via the Meissner effect, the magnetic field is partially screened from the regions outside the vortex cores. Now the system will consist of, alternating regions of superconducting material with a screened magnetic field, and the vortices where most of the magnetic field resides. As discussed in Ouyed et al. (2004), this has interesting consequences on how the surface magnetic field adjusts to the interior field which is confined in the vortices. In Ouyed et al. (2005) we performed numerical simulations of the alignment of a quark star's exterior field, and, found that the physics involved was indicative of SGR activity[1].

It has been shown that pure CFL matter is rigorously electrically neutral despite the unequal quark masses (Rajagopal & Wilczek 2001). However other work (Usov 2004; and references therein) indicates a thin crust ($M_{\rm crust,max} = 10^{-5} M_\odot$) is allowed around a quark star due to surface depletion of strange quarks. In our model we have assumed no depletion of strange quarks which implies a bare quark star. Another simplicity of our model resides in the fact that we have a single superconducting fluid (the CFL phase). In the case of neutron stars (e.g. Konenkov & Geppert 2000), one has to deal with the neutron superfluid inducing the neutron vortices parallel to the rotation axis and the proton superconductor inducing the fluxoids (the magnetic field concentrated into quantized proton vortex lines) in a direction parallel to the magnetic field. In our case, having a single superconducting-superfluid implies that fluxoids are contained inside the vortex cores.

Thus, the forces at play are quite different for the CFL quark star than for a neutron star: i) the drag force induced by electron scattering is non-existent in our model since no electrons are admitted in pure CFL matter; ii) in the case of neutron stars, there exists a force on the vortices due to the variation of the neutron or proton superfluid gaps (Δ) with density which can expel or trap the vortices (Hsu 1999). In quark matter, the variation of the gap is not well constrained and it is common to assume the BCS relation $\Delta \propto \sqrt{1-(T/T_c)^2}$. The nearly uniform temperature implies a nearly uniform gap inside the quark star, thus, no vortex trapping; iii) the absence of a crust removes all possible surface pinning of the vortices and the fluxoids (this also implies no Magnus force). Even in a thin crust case the

[1]See simulations: www.capca.ucalgary.ca/~bniebergal/meissner/

superconducting matter and thus the vortices and fluxoids do not extend into the crust, which is suspended $\sim$ 100-1000 fermi above the surface of the star (Alcock et al. 1986). We argue that crustal pinning can be neglected in this case too. iv) The remaining force is the buoyancy force, which is not counteracted by any pinning forces. Thus the spin-down determines the rate of vortex expulsion.

3. Magnetic Field Decay and Spin Evolution

Following the initial alignment event, is the quiescent spin-down phase where the outermost vortices are pushed to the surface and expelled (Ruutu et al. 1997). The magnetic field contained within the vortices is also expelled and annihilates through magnetic reconnection events near the surface of the star causing energy release presumably in the X-ray regime. The number of vortices decreases slowly with spin-down leading to continuous, quiescent energy release which can last until the magnetic field is insufficiently strong to produce detectable emission.

In the aligned-rotator model the star spins-down by magnetospheric currents escaping through the light cylinder. For a neutron star, these currents originate in the crust. Instead, in our model, pair production from magnetic reconnection[2] supplies the currents. The approximation for an aligned rotator Is (i.e. see Mészáros 1992),

$$\frac{\dot{\Omega}}{\Omega} \approx -\frac{B_{\rm QS}^2 R_{\rm QS}^6 \Omega^2}{Ic^3}, \tag{1}$$

where I is the quark star's moment of inertia. The vortex annihilation rate, given in Ouyed et al. (2004; Eq. 22), and Eq. 1 are then solved simultaneously to give the magnetic field decay,

$$B_{\rm QS}\left(t\right) = B_{\rm QS,0}\left[1+\frac{t}{\tau}\right]^{-\frac{1}{6}}, \tag{2}$$

and period evolution,

$$P_{\rm QS}\left(t\right) = P_{\rm QS,0}\left[1+\frac{t}{\tau}\right]^{\frac{1}{3}}. \tag{3}$$

[2]Since our model does not allow for a crust or electrons, the parallel electric field induced by the potential drop along the magnetic field lines (as happens in a neutron star; e.g. Mészáros 1992) is insufficient to remove charges from the quark star. Instead the current necessary for the braking torque is provided by the e^+e^- pairs generated by magnetic reconnection events. First estimates show that for the magnetic fields in our model, pair production provides sufficient current to remove angular momentum from the star.

Where $B_{\rm QS,0}$ is the initial magnetic field strength immediately after the burst at the surface of the quark star, $P_{\rm QS,0}$ is the initial period, and $R_{\rm 10,QS}$ is the quark star radius in units of 10 km. We have also defined a characteristic age (in years) of,

$$\tau_{yr} = 5\times 10^4 \left(\frac{10^{14}\ {\rm G}}{B_{\rm QS,0}}\right)^2 \left(\frac{P_{\rm QS,0}}{5\ {\rm s}}\right)^2 \left(\frac{M_{\rm QS}}{M_\odot}\right) \left(\frac{10\ {\rm km}}{R_{\rm QS}}\right)^4 . \tag{4}$$

Eqs. (2 & 3) naturally produce the field decay behavior sought by Colpi et al. (1999), who used a phenomenological power law to describe different avenues of magnetic field decay. However, because we solve period and magnetic field simultaneously, our model predicts a different period evolution. Colpi et al. (1999) also argue that some efficient mechanism of magnetic field expulsion from the star's interior must exist in order to explain a short field decay timescale. Our model provides a natural explanation since the absence of a crust allows efficient expulsion of unpinned vortices.

In our case, as illustrated in Figure 1, both the period and magnetic field remain unchanged to within a factor of two for 5×10^5 to 5×10^7 yrs given a quark star radius of 10 km and field strengths typical of AXPs/SGRs in the quark star model (10^{13} - 10^{14} G). Therefore we have shown that, if we presume long-term period evolution is dictated by magnetic field decay, then the periods of AXPs/SGRs will remain close to their initial periods for timescales relevant to observations. Thus, our model may provide answers to issues raised by Psaltis & Miller (2002), and, observations of variable braking indices in AXPs/SGRs made by Kaspi et al. (2000). Variable braking indices in our model would most likely be due to an unsteady magnetic field decay which is beyond the scope of this work. Psaltis & Miller (2002) also argue that the final periods of AXPs/SGRs will be no greater than ~ 12 s, thus, AXPs/SGRs that are born in a narrow range of periods should remain so indefinitely. However, they omitted magnetic field decay in their stochastic calculations. Our results, based on field decay, indicate that the final periods may indeed be larger but only significantly after timescales of 5×10^5 to 5×10^7 years (for $10^{13} < B < 10^{14}$ G; see Figure 1). However at these timescales, the likelihood of observing objects with periods greater than 12 s is low because of the significant decay in field strength, and dependence of luminosity on B^4 (Ouyed et al. 2004; Eq. 24). Consequently our model predicts that, since AXPs/SGRs are born within a narrow range of periods, they will remain so for at least 10^6 yrs depending on how compact the star is, after which the range will broaden slightly over time. This will be discussed in detail in Section 5.

4. Radio Quiet Isolated Neutron Stars

Radio quiet isolated neutron stars (RQINSs) are a class of older ($\sim 10^6$ yrs) stars possessing strong magnetic field strengths (10^{13} to 10^{14} G) and exhibit a clustering in their observed periods similar to that of AXPs and SGRs. RQINSs have previously been speculated to be related to AXPs and SGRs (see Treves et al. 2000 for a review), however, using our model we will describe how RQINSs are a natural consequence of the magnetic field decay due to vortex expulsion in quark stars.

Firstly RQINSs exhibit no radio pulsations, which in our model is a necessary consequence of the AXP/SGR burst which causes the magnetic field to align with the star's rotation axis. Furthermore after the quark star's field has aligned, it will spin-down through magnetic braking as described in Eqs. 2 & 3, and, for ages of the order of $\sim 10^6$ yrs, we arrive at results indicative of RQINSs. Specifically, if a quark star experiencing an AXP/SGR burst is born with a period of $P_0 = 5$ s and magnetic field strength of $B_0 = 10^{14}$ G, then by the time it reaches ages estimated for RQINSs it will have attained a period of 11 s and it's field will have decayed to $\sim 5 \times 10^{13}$ G. This is illustrated in Figure 1. Here, the decrease in field strength by a factor of two results in a decrease in luminosity by a factor of 2^4 (see Ouyed et al. 2004; Eq. 24), which suggests that only RQINSs possessing an initially strong field are more likely to be detectable.

4.1. Evolution in the P-$\dot{P}$ Diagram

Figure 2 describes the period evolution of a recently born quark star on a $P - \dot{P}$ diagram for various initial surface magnetic field strengths. The period derivative, in our case of a magnetic field decaying through vortex expulsion, is attained from Eq. 3. The parameters selected for the quark star in Figure 2 are, a mass of $1M_\odot$, radius of 10 km, and initial period of 5 s. Increasing the radius will shift the evolutionary tracks upwards, whereas changing the mass has little effect, and, selecting different initial periods shifts the tracks left or right. So, with expected quark star parameters, all RQINSs follow evolutionary tracks backwards to the region in the P-$\dot{P}$ diagram suggestive of AXPs/SGRs. More specifically, Figure 2 shows that the source indicated by the small square (RXJ0720.4-3125) is $\sim 5 \times 10^5$ yrs old and has an initial field strength of $\sim 10^{14}$ G, given the observed period of 8.391 s and period derivative of $\sim 1.5 \times 10^{-13}$s s^{-1}.

Figure 2 also shows two RQINSs which are marked by down-arrows, indicating that only upper-limits on $\dot{P}$ are known (Pons et al. 2005). In the context of quark stars, this translates into upper-limits on the field strengths of the order of 10^{15} G, and a lower-limit

on their age of $\sim 10^4$ yrs.

5. Compactness and Period Clustering

Although there may be an insufficient number of observed RQINSs to conclude definitely the exact range of periods they are clustered into, Pons et al. (2005) shows that all the periods of observed RQINSs so far are between the same range as AXPs and SGRs (3-12 s). This concurs with our results in that, after 10^6 yrs, only highly compact stars will have periods which do not deviate much from the range of their progenitor AXPs/SGRs (see Figure 3). The standard neutron star model for AXPs/SGRs spinning down due only to dipole radiation has $\dot{P} \propto B^2 R^4$, which negates the possibility of period clustering after 10^6 yrs. In our model, the magnetic field is expelled from the star's interior and so decays in time, which in turn decreases the spin-down rate causing any initial clustering in periods to remain. Also, Eq. 3 predicts that only very compact stars can retain this clustering for timescales of the order of 10^6 yrs. This can be understood physically by realizing that, given a magnetic field strength and period, a more compact star will have a higher magnetic energy density in each vortex. This causes each vortex expulsion event to remove greater amounts of magnetic field from the system, making magnetic braking become increasingly more ineffective.

The observed periods of RQINSs are indeed clustered, however the range of this clustering and the mean on which it is centered is inconclusive. Upon detection of more RQINSs, we will be able to conclude more confidently whether the mean period of RQINSs is higher than that of AXPs and SGRs, and, this in turn will allow us to predict more accurately the radius of quark stars.

6. Conclusion

We have shown in this Letter that, if CFL quark stars are born with periods and magnetic fields characteristic of AXPs/SGRs, then both period and field will remain unchanged within a factor of two for timescales of the order of 5×10^5 to 5×10^7 yrs (Figure 1). Therefore, because AXPs/SGRs are born within a narrow period range, their periods will remain clustered for timescales applicable to observations. Moreover, after timescales of 10^5 to 10^6 yrs their periods will be on average higher. However, because only a relatively small number of RQINSs have been discovered to date, it is difficult to determine whether their period mean is indeed higher than that of AXPs/SGRs.

Also, within the context of the quark star model, we have used a $P - \dot{P}$ diagram

(Figure 2) to illustrate how the field strengths and periods of AXPs/SGRs evolve in a manner indicative of RQINSs. Considering the quark star's magnetic field becomes that of an aligned dipole, our model provides a natural explanation as to why no radio pulsations are observed in RQINSs.

Finally, from Figure 3, we have shown that the compactness of a quark star is related to period clustering. Only highly compact objects are able to maintain clustering in their periods for timescales of 10^6 yrs, suggesting that the observed clustering in RQINS periods is an indicator of a compact quark star progenitor. Further detections of RQINSs will allow us to determine exactly how compact these progenitors are.

We thank K. Mori for his insightful discussions. This research is supported by grants from the Natural Science and Engineering Research Council of Canada (NSERC).

REFERENCES

Alcock, C., Farhi, E., & Olinto, A. 1986, ApJ, 310, 261

Colpi, M., Geppert, U., & Page, D. 2000, ApJ, 529, L29

Hsu, S. D. H. 1999, Physics Letters B, 469, 161

Kaspi, V. M., Lackey, J. R., & Chakrabarty, D. 2000, ApJ, 537, L31

Keränen, P., Ouyed, R., & Jaikumar, P. 2005, ApJ, 618, 485

Konenkov, D., & Geppert, U. 2000, MNRAS, 313, 66

Mazets, E. P., Golenetskii, S. V., Ilinskii, V. N., Aptekar, R. L., & Guryan, I. A. 1979, Nature, 282, 587

Mészáros, P. 1992, University of Chicago Press, c1992

Meissner, W., Ochsenfeld, R., 1933, Naturwiss., 21, 787

Ouyed, R., Dey, J., & Dey, M. 2002, A&A, 390, L39

Ouyed, R., Elgarøy, Ø, Dahle, H., & Keränen, P. 2004, A&A, 420, 1025

Ouyed, R., Niebergal, B., Dobler, W., Leahy, D., 2005, arXiv:astro-ph/0510691.

Pons, J. A., Pérez-Azorín, J. F., & Miralles, J. A. 2005, Memorie della Societa Astronomica Italiana, 76, 518

Psaltis, D., & Miller, M. C. 2002, ApJ, 578, 325

Rajagopal, K., & Wilczek, F. 2001, Phys. Rev. Lett., 86, 3492

Ruutu, V. M., Ruohio, J. J., Krusius, M., Plaçais, B., Sonin, E. B., & Xu, W. 1997, PRB, 56, 14089

Srinivasan, G., Bhattacharya, D., Muslimov, A. G., & Tsygan, A. J. 1990, Current Science, 59, 31

Tilley, D. R., & Tilley, J. Superfluidity and Superconductivity, 3rd ed., Bristol, England, Hilger 1990

Treves, A., Turolla, R., Zane, S., & Colpi, M. 2000, PASP, 112, 297

Usov, V. V. 2004, Phys. Rev. D, 70, 067301

This preprint was prepared with the AAS LaTeX macros v5.2.

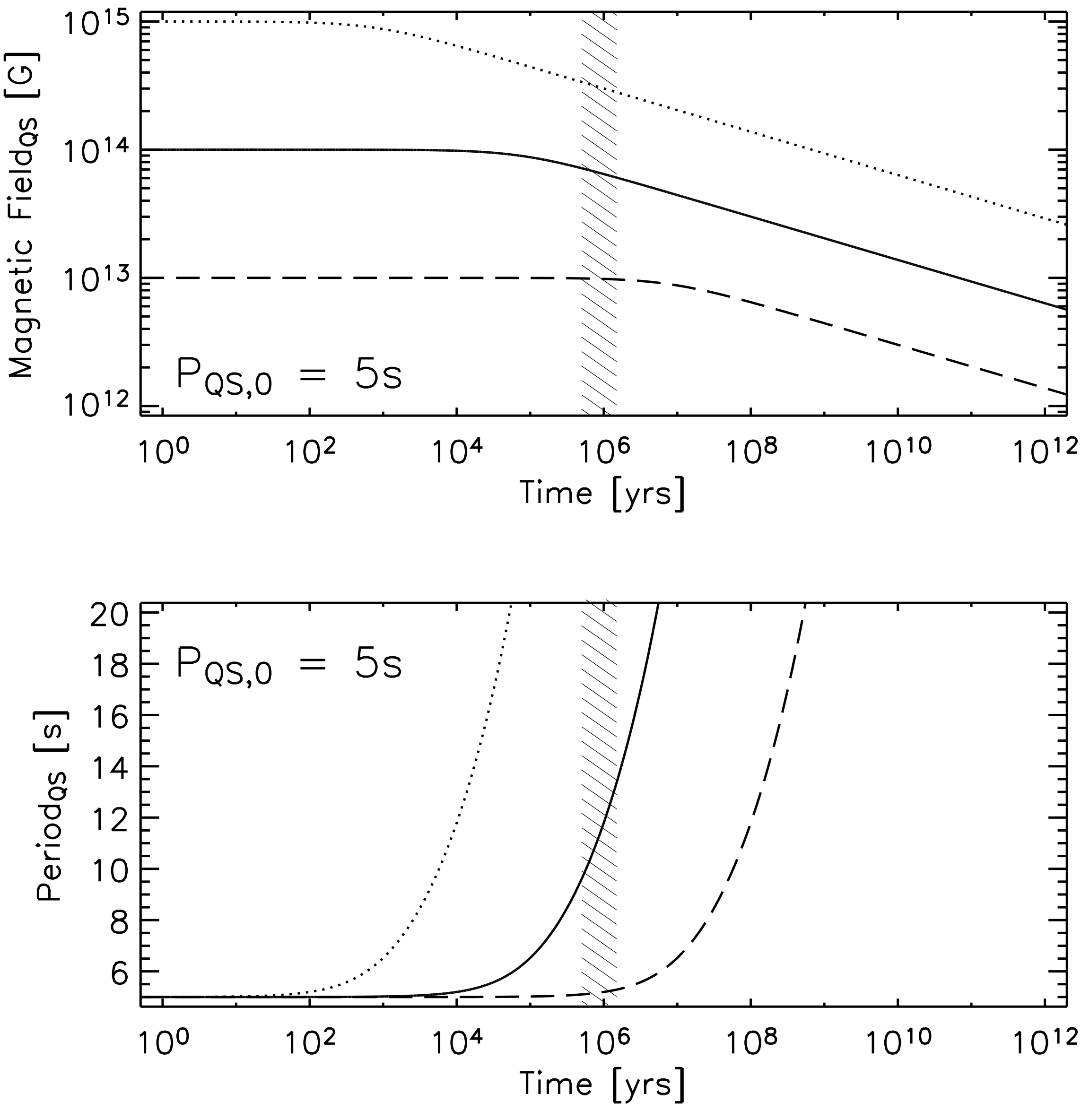

Fig. 1.— Magnetic field decay and spin-down for three different values of initial field strength (10^{13}, 10^{14}, and 10^{15} G). The quark star has an initial period of 5 s and a radius of 10 km. The dashed region represents the only two RQINSs having an inferred age.

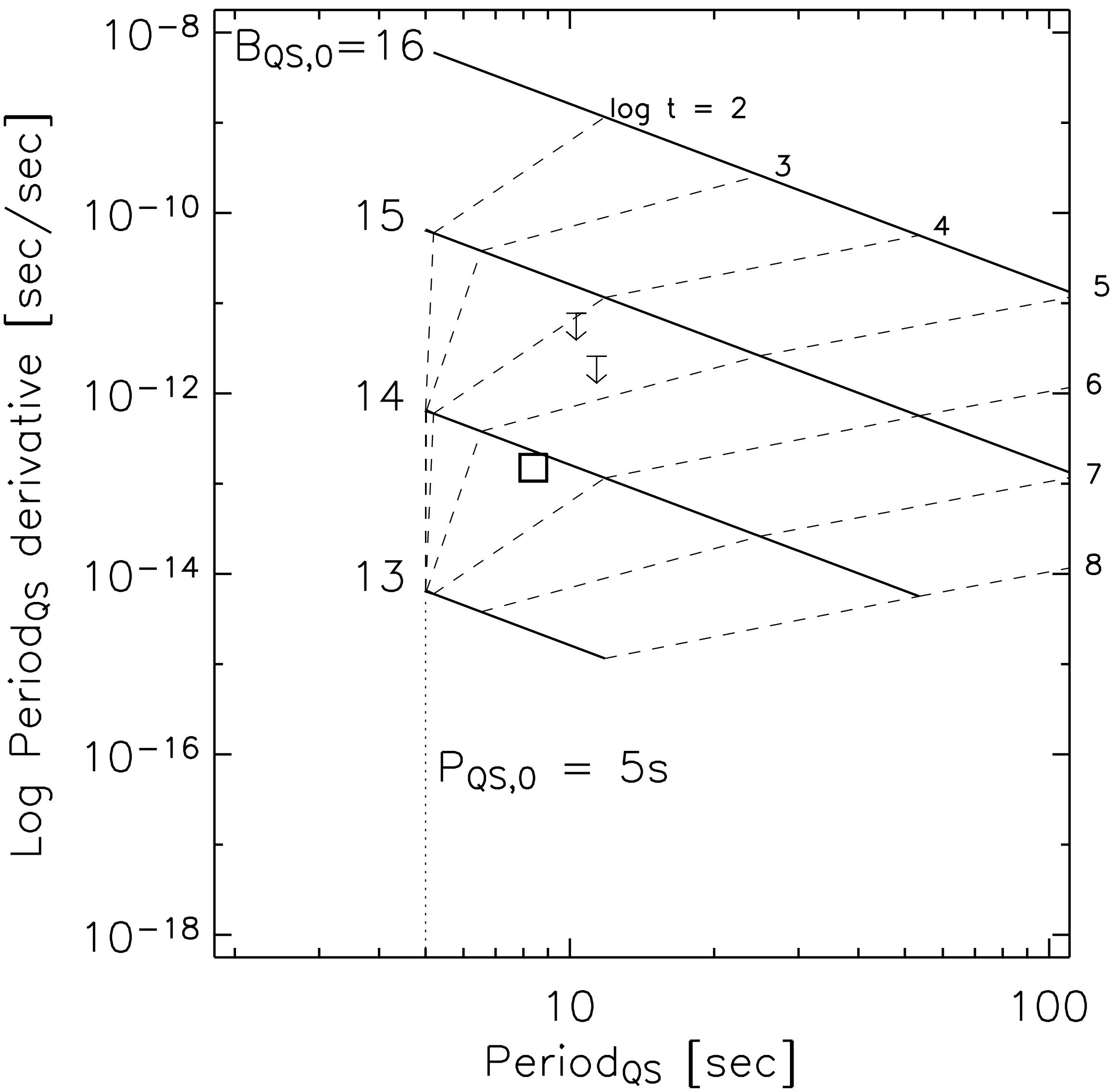

Fig. 2.— Evolutionary tracks (solid lines) for differing surface magnetic fields as indicated, and an initial period of 5 s. The quark star is assumed to have a 10 km radius and a mass of 1 $M_{\odot}$. RQINSs are marked with the small box or down arrows (for sources with only an upper limit on $\dot{P}$). Dashed lines represent time in years from the birth of the quark star. All evolutionary tracks lead to birth parameters indicative of AXPs/SGRs.

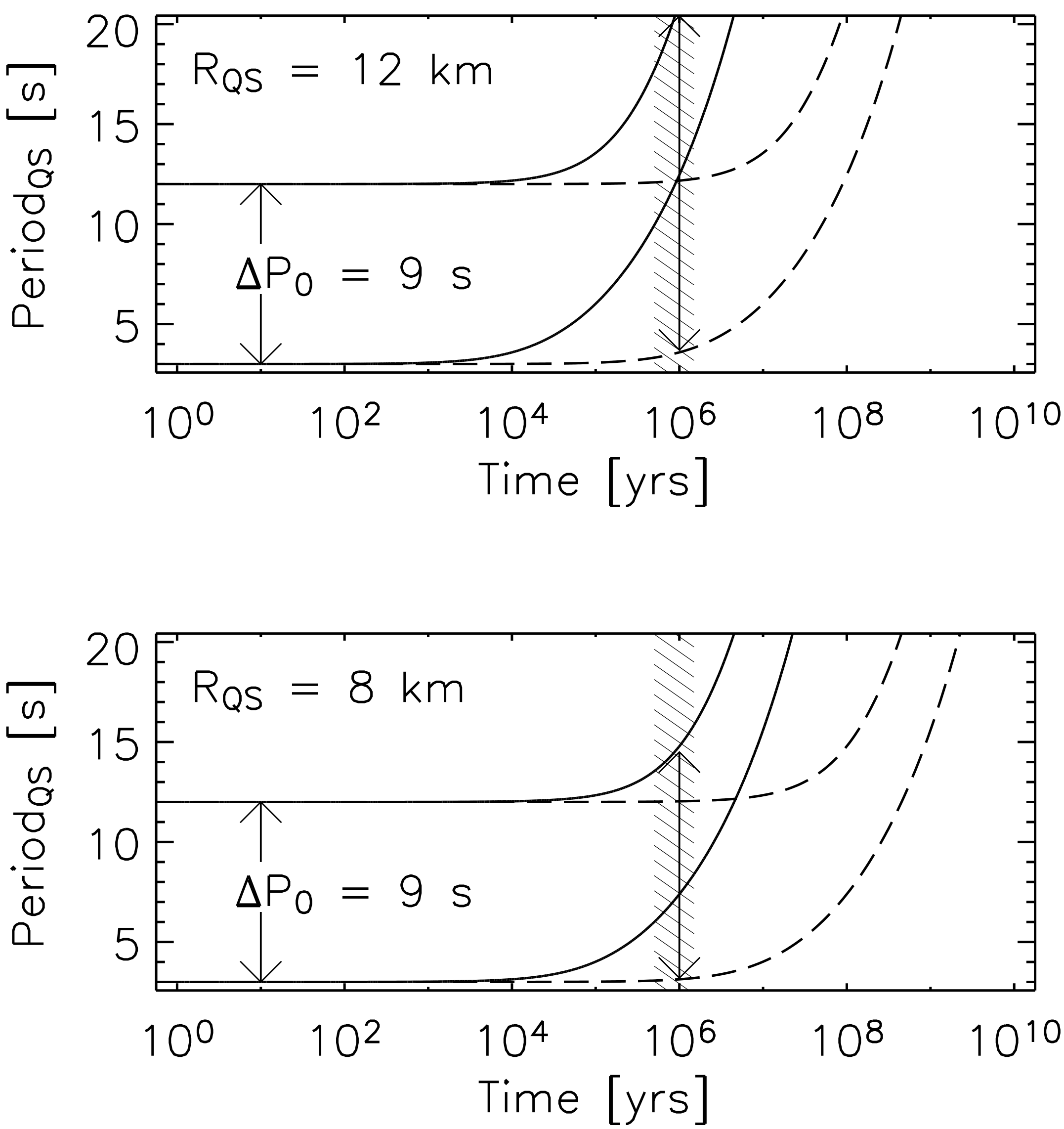

Fig. 3.— Period clustering evolution for initial magnetic fields of 10^{13} G and 10^{14} G (dashed and solid lines respectively), for radii of 12 (upper panel) and 8 km (lower panel). The upper and lower curves in each panel are the maximum and minimum of the period range. If $R_{\rm QS} > 10$ km, then after 10^6 yrs the period range becomes large, suggesting only compact objects undergoing vortex expulsion can explain period clustering in RQINSs.